DOCUMENT RESUME

ED 334 073 SE 052 159

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TITLE Learning To Talk Mathematics.
SPONS AGENCY National Science Foundation, Washington, D.C.
PUB DATE Apr 91
CONTRACT MDR-885-0560
NOTE 25p.; Paper presented at the Annual Meeting of the American Educational Research Association (Chicago, IL, April 3-7, 1991).
PUB TYPE Reports - Research/Technical (143) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Classroom Communication; Classroom Techniques; *Discussion (Teaching Technique); *Elementary School Mathematics; Grade 3; Higher Education; *Learning Strategies; Mathematical Enrichment; Mathematics Education; *Mathematics Instruction; Primary Education; Problem Solving; Qualitative Research; Teaching Methods
IDENTIFIERS *Discourse

ABSTRACT

Calls for increased student involvement in mathematics classroom learning situations are due primarily to the recognition that a traditional lecture/demonstration format within school mathematics instruction is not effective in fostering and promoting students' problem-solving abilities, mathematical reasoning power, and mathematical communication skills. In an attempt to come to a better understanding of mathematics classroom learning, a study focused on the particular activity of mathematical discourse, with the main goal being both the characterization of the potential learning opportunities and the inference of students' construction of mathematical meanings. The participants in this study were the students and teacher in a third-grade class of a laboratory school associated with a Southern university, chosen to reflect the gender, socioeconomic status, ability levels, and ethnicity of the State. Various qualitative research techniques such as participant observations, interviews, and video recordings were used to collect data, which were categorized according to style and meaning of class participation. Results indicate that classroom mathematical discourse is a rich environment for both student growth and mathematical learning, which was confirmed by more elaborate and integrated schema for mathematical reasoning and problem solving. (22 references) (JJK)

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Learning to Talk Mathematics

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Paper presented at the 1991 meeting of the American Educational Research Association, Chicago.

The research in this paper was supported by the National Science Foundation under grant No. MDR 885-0560 and by the Florida State University through Dissertation Fellowship. The opinions expressed do not necessarily reflect the views of these funding agents.

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The National Council of Teachers of Mathematics (1989) suggests that problem solving, mathematical reasoning, and mathematical communication should be an integral part of mathematics instruction at all grade levels. It has been recognized that a traditional teacher demonstrating-students practicing type of instruction is not effective in achieving these goals, and calls for student involvement in mathematics learning have appeared in all major educational reports (e.g. National Council of Teachers of Mathematics,1989; National Research Council, 1989). Various alternative instructional strategies like using manipulatives, conducting group projects and engaging in class discussions have been suggested, yet using these strategies do not necessarily facilitate mathematics learning. The effectiveness of any instructional activity depends on how, why and when the activity is used as well as the beliefs and social norms of the class. A better understanding of mathematics classroom practices is needed in order to give empirical support for educational reforms (Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti and Perlwitz, 1991; Cobb, Yackel, Wood and NcNeal, in press; Lo, Wheatley, and Smith, 1990).

In an attempt to understand classroom learning, we have focused our attention on a particular activity, class discussion. Researchers and teachers have attempted to give definitions of the term "mathematics discussion" (Pirie & Schwarzenberger ,1988; Smith, 1987). In our study, the term "mathematics class discussion" refers to a classroom activity with the following characteristics, 1) student interpretations of a mathematical task is the focus of the activity, 2) student-to-student interaction is encouraged, and 3) the teacher's role is to facilitate student-to-student communication rather than to guide or evaluate. Furthermore, we have limited our data collection to those mathematics class discussions which were conducted after small group problem solving activities. We were fortunate to be involved in a classroom which used small group problem solving and whole class discussion as a basis for daily mathematics lessons, thus providing an opportunity to study class discussions.

The main goal of this study was to characterize the potential learning opportunities and to infer student constructions of mathematics meaning in class discussions. To achieve this goal, we felt it was crucial to understand the social dynamics of class discussions within which mathematics

meanings were negotiated and constructed. Our analysis showed that students and the teacher had to face two challenges as they attempted to talk mathematics as a group. The first challenge came from the need to reflect on one's own problem solving activities and to express those thought processes in language. The second challenge came from the need to communicate one's mathematics ideas with other members in the class. In this paper, we will use two class discussion episodes to illustrate these challenges as well as the growth the class made from the beginning to the end of the school year.

Theoretical Framework

In any study, what is observed and reported is profoundly influenced by the researcher's theoretical orientation (Sless, 1986). Therefore it is important for us to provide this information to the readers. From a constructivist's point of view, learning occurs when an individual adapts his/her functioning schemes to neutralize perturbations that arise through interactions with our world (Steffe, 1990). The interactions with other people frequently provided a source of perturbation for the developing child (von Glasersfeld, 1989). The source of perturbations, problematic situations, can take a variety of forms in class discussion. In a discussion setting, speakers may face the challenge of reflecting on their problem solving activities, re-presenting their methods in verbal statements, interpreting audience's responses and attempting to communicate. We do not expect students who are confronted with alternative ideas to immediately modify their functioning schemes because the ideas may not become problematic. Class discussion can provide potential opportunities for students to reflect and elaborate on their existing schema (Barnes, 1976; Bruner, 1986; Duckworth, 1987).

It is important to recognize that mathematics knowledge is social knowledge (Balacheff, 1990; Solomon, 1989) and the mathematics class forms a community (Balacheff, 1990; Schoenfeld, 1988). Therefore, knowing a mathematical concept includes knowing how other people think about that concept and how they expect to see this concept used. In the setting of class discussion, different ideas are presented and discussed. For the class to function smoothly as a group, there is a need to negotiate social norms and taken-to-be-shared mathematical meanings.

Class social norms not only influence students' and the teacher's expectations of their and each other's responsibilities as a member of the community, they also influence students' and teachers' beliefs about mathematics and learning. Cobb and his colleagues (1991) have shown that students in those classes which attempted to negotiate social norms like "explaining and justifying solutions, trying to make sense of explanations given by others, indicating agreement and disagreement, and questioning alternatives in situations where a conflict between interpretations or solutions has become apparent" (p.7), held the belief that the success for learning mathematics relied on developing their own methods rather than using those methods shown by the teacher.

Participants and Setting

The participants in this study were the students and teacher in a third grade class of a laboratory school associated with a Southern university. The population of the school is chosen to reflect the gender, socio-economic status, ability, and ethnicity of the State. The teacher in this study used small groups and whole class discussion as an instructional strategy in her mathematics classes during the school year prior to the commencement of this study. The students of this study also had similar instructional experiences. Except for three transfer students, most students knew each other although they came from two different second grade classes. The researchers had visited the second and third grade classrooms frequently during the year before this study began and good rapport had been built through those interactions.

The classroom was furnished with several large tables instead of individual student desks. Each table could sit from two to six students depending on its size and shape. The tables were arranged so as to leave a space in front of the blackboard for students to assemble on the floor for class discussions. This was done partly because the teacher felt the need to gather students around when having a class discussion and also to facilitate video recording. Some students sat in chairs around the periphery of the groups while other students sat on the floor. On a given day, the students might be clustered together, spread apart or composed of up to three distinguishable rows.

Unlike commonly seen teacher-led didactic discussions which are an adjunct to exposition, the purpose of class discussions in this classroom was for students to explain their solution

methods to problems previously solved and to negotiate mathematics meaning. In other words, students' methods were the focus of the discussion, not the teacher's methods. The role of the teacher was to facilitate the verbal interactions among her students and not "lead" students to previously determined mathematical statements. In no case did she evaluate student answers or choices of solution methods. Even when students could not reach a consensus on a task after a discussion of fifteen minutes, she would still not resolve the conflict by taking a position on the dispute. Instead she posted the task on the board and encouraged students to think more about it .

Data and Analysis

The data of this study were collected from the beginning of the school year until the first week of May. Various qualitative research techniques such as participant observations, interviews, and video recording were used to collect data. The length of each discussion ranged from ten to twenty-five minutes. Classroom dialogues which seemed to be helpful in analyzing related issues were transcribed, and became part of the data. These notes of video recordings, together with field notes and interview records formed the text-based data for further analysis. In an attempt to construct an explanation of the children's actions in this setting, conjectures were formed when reading these text-based data and these conjectures were then subjected to refutation and modification as new information was gathered from subsequent viewing of video recordings and discussion among the researchers. Instead of using predetermined categories to record student participation, the meaning of this participation and the different styles of participation were determined as the data were analyzed.

In this paper, we will use two classroom events to illustrate the problematic situations students faced as well as the growth they mad during the school year. The first episode occurred on September 26, 1989. By that time, the class had engaged in class discussions for fifteen lessons. Social norms and interactive patterns were in the process of being established, although subject to change. The second episode occurred on April 27, 1990, five weeks before the end of school year. During both episodes, the class engaged in solving word problems. The task discussed in September was as follows,

I pay 18 cents for a newspaper. I sell the paper for 25 cents.
How much do I make on each paper I sell?

The task discussed in April was the following,

Diana bought a plant for $2.75, then she sold it for $3.49. How much money did she earn by selling the plant?

These two classroom discussions were selected because 1) class interactions of these two episodes were representative of those at the beginning and near the end of school year; 2) comparable mathematical tasks were discussed. Our analysis will focus on comparing and contrasting the interactive patterns and underlying social norms, as well as the student explanations and mathematics constructions.

September lesson

On this day, Jeff was the first student who was chosen by the teacher to explain his method. He happened to work by himself that day, normally he would have had a partner in small group problem solving.

Jeff:	See, he had to spend the 18 cents to get the newspaper
Jenny:	What's your answer?
Melvin:	What's your answer ?
(some laughter)	
June:	Please (Teacher: Jenny.) do not interrupt him like that.
Teacher:	Jenny, is that a really good manner to interrupt other people in the middle of the explanation? It sounds a little rude to me.

Jeff was unsettled by the unexpected interruption. He appeared to be lost. Students began to talk among themselves. The teacher asked students to be patient. One student passed a paper to Jeff. Jeff looked at it, paused a while, then finished his explanation, "I counted from..18 to get... It's just the difference between 18 and 25, so I counted from 18 to 25 then I got the 7."

Initially Jeff attempted to give a description of how he interpreted the task, but he was unable to do so. After struggling for 27 seconds, he concluded his explanation as though the task was an arithmetic computation rather than a word problem. The interaction between Jenny and June can be seen as a conflict of incompatible expectations of the speaker's explanation. Jenny interrupted

Jeff's explanation because she expected Jeff to state his answer before giving the explanation. Melvin, who shared the same expectation with Jenny, supported her question. However, June felt that Jenny's action was inappropriate and thus this intervened. The teacher's final statement, although being addressed to Jenny, was really a reminder to the class. She expected her students to show respect for each other.

After the teacher repeated the last sentence of Jeff's statement, she asked for volunteers to present another way. Jenny and her partner Mary were selected to be the next speakers.

Jenny: See, what we did...we got 7 too. At first, we didn't, I didn't understand because I didn't understand that the person needed to pay 18 cents for the newspaper. And Mary explained it to me and I understood. Then I took 18 away from 25 and I got 7.

Teacher: Mary, would you tell us what you explained?

Mary: Jenny didn't-

Teacher: Mary, wait until Casey gets herself turned around and is focusing on you, please.

Mary: Jenny didn't understand what "I pay 18 cents for a newspaper" meant. And I had to explained this to her. I said "Jenny, think of it this way. You know when you buy a packet of gum at store, you pay them inyour money then you take the pack of gum. Then eventually she understood.

Jenny: That's the same, It's the same with the newspaper. She said you buy...you have to pay, you have to pay 18 cents for the newspaper. People give you 25 cents for the newspaper, and the 18.. I took , we took 18 away from 25, and we got 7.

Jenny's initial statement showed that she was not sure which part of her problem solving experience should be included in her explanation. She felt that she had to say more about "took 18 cents from 25 cents", and her explanation had to be different from Jeff's. Therefore she stated the confusion she had and how this difficulty was overcome (with Mary's explanation). She did not feel the need to describe the process. The teacher then asked Mary to "tell us what you explained". With this intervention, Jenny's and Mary's explanation was more the details of their interpretations

of this particular task. The teacher's request could be seen as an attempt to negotiate what needed to be included in an explanation. Although it was not clear how Mary's explanation might help Jenny to understand this task, the teacher felt that other students might learn something from a more detailed account of the problem solving process Jenny and Mary used. Notice that Jenny had changed from "I" to "we" twice in her statements. This suggested that Jenny was trying to coordinate both Mary's thoughts with her own; she was also aware of the obligation to explain "how Jenny and Mary solved this task", not just her own method.

The teacher had observed students solving this task prior to the discussion. Therefore she had anticipated the difficulty which many of her students experienced in small groups. So she decided to focus the discussion on that particular point.

Teacher:	Well, if, if somebody paid 25 cents for a newspaper, why can't he say he made 25 cents on each newspaper?
Students:	Yes, why?
Jenny:	Well, because see...I sort of ...I don't...um, I am caught in between about that question, because see...um, what I thought was, um, he had to use part of that 25 cents to buy the newspaper from the company to sell to the other people.
Teacher:	How many people agree with what Jenny just said?
Students:	Agree. Disagree. I don't understand. I understand it pretty well.

The teacher's question clearly influenced Jenny to re-examine her position on this particular task. Jenny's answer showed her struggle as well as the increase control she had over her own thoughts. As students' opinions were evenly divided between two different interpretations, Jenny was ready to face the challenge from the students who had different viewpoints. She called on June.

June:	What, what, what do you try, what do you try to explain to us? I don't know, like, I don't understand what you are trying to say.
Jenny:	What don't you understand?
June:	I don't understand. You are mumbling.
Jenny:	Okay, What I said-
Teacher:	I think the problem is these people sitting over here are mumbling to each other. Casey, you come sit right here please.

Jenny: See, Mary, people pay in 25 cents right ?

June: Right.

Jenny: Good. He has to take 18 cents away from the paper, newspaper-

June: What do you mean by-

(Discussion was interrupted by many students' comments)

Initially June indicated that she could not understand Jenny's explanation. Naturally, Jenny asked June to provide more information as to which part of her explanation needed to be elaborated. However, June was unable to form a specific question, instead she accused Jenny of mumbling. Jenny interpreted June's statement as an indication that June did not hear her explanation, so she tried to explain again. This time she tried to make sure June heard and understood every part of her explanation . But June interrupted.

June was unable to finish her question because many students attempted to comment on the on-going events. Some students blamed June for not paying attention; Some students tried to help establish the order; other students asked questions about Jenny's explanations. Jenny tried to get others to listen and indicated that she had only called on June (Officially, only Jenny, June and June had the right to talk). June and Jenny tried to continue the discussion but were repeatedly interrupted by other students. The classroom was noisy for one minute and ten seconds before the discussion between Jenny and June could resume.

June: What are you trying to say? You said you have to take 18 from 25?

Jenny: Yes!

June: Why do you have to take 18 away from 25?

Jenny: Because he has to that... Because he can't own the newspaper himself to go sell to the other people, because he has to buy it from the company that made the newspaper.

June: Why, why do you <u>really</u> have to take away because the paper is really 25 cents.

Jenny: I know that.

June: Then why do you have to take 18 cents away from 25?

Jenny: (upset) Oh wait, you are not listening to me because I have said this <u>over and over</u> again. (with Mary) He takes 18 cents away from 25 to buy the newspaper from the company, not the people.

34 June: What company?

35 Jenny (with Mary) The company that made the newspaper.

36 June: I don't understand.

(Discussion was interrupted.)

Both Jenny and June were frustrated at this point, as were those students who attempted to express their opinions but were not given opportunities to do so. During the one minute and ten second noisy period, Mary, Jenny and June basically had the same conversation as the one above. That was why Jenny and Mary were so upset about having to say the same thing over and over again. The fact that June asked the same questions over and over again indicated to her that June was not listening to her explanation at all. However, June thought that she had heard every word of Jenny's explanation and she had made her point clearly. It was Jenny who did not give sufficient clarification. As the tone of Jenny's and June's voices changed, Mary's voice changed also. Communication became increasingly.

June and Jenny had two other unsuccessful attempts to communicate to each other before the teacher finally intervened. The teacher explained the role of newspaper company in the buying and selling newspapers, because she felt that was the source of student confusions. However, June and Jenny resumed their debates as soon as the teacher finished her explanation. Finally, the teacher terminated the discussion.

In the September lesson, the class had not worked out a set of social norms to help them function smoothly in mathematics discussion, and students had difficulty in both putting their ideas in language and communicating with each other. Problematic situations existed when students attempted to figure out how to describe their problem solving activities; when student had incompatible expectations of an clear explanation; when students accused each other of not doing what they were supposed to do. Although June was a central figure in this discussion, her interpretation of the task was unclear; she did not understand why Jenny had to take eighteen cents away. Before the teacher asked Mary to elaborate on Jenny's explanation, little student interpretations of this task could be inferred. Even though the teacher had told students that it was the method rather than the answer which was important in mathematics discussion, some students

had not taken that as their personal goal and also there was a need to negotiate what counted as a clear explanation.

April lesson

On this day, Brad and his partner were the first group chosen to explained their method. Brad first read the statement of the task, then gave his explanation.

Brad: This is what we thought. it says she bought the plant for two dollars and seventy-five cents, then.. so.. then she...Okay...then she went back home, and then one of her friends wanted this plant for three dollars and forty-nine cents. So she sold it to her, then she got three dollars and forty-nine cents. And then it said "How much money did she earn by selling the plant?" so, we said three dollars and forty.. forty-nine cents.

Students: Agree. Disagree.

The teacher pointed to Jeff, indicating that it was his turn. Other students wanted to share too and were noisy. The teacher gestured for these students to be quiet.

Teacher: Everybody is going to have a chance. Just remember what you are going to say.

Jeff: Because...see...I think it's a dollar or something. See, it says she got it for two dollars and seventy-five cents, she sold it for three dollars and forty-nine cents, so she...so she, it meant how much did she get more.

Both Brad's and Jeff's explanation had changed from "tell the answer" to "why they thought their interpretations were valid." Jeff did not even care what the correct answer was. He assumed other students would understand what he meant even though he did not give the exact amount. Notice also that Brad had personalized the task by putting it into a story format. There was no "friend wanted his plant for three dollars and forty-nine cents" in the original task. He felt that this short "add-on" was an important part of his understanding of the problem, and he felt comfortable sharing it with his classmates.

Teacher: Somebody want to add on to that?

Brad: It says how much money did she earn.

Brad's comment was given without the teacher's permission and after the teacher had pointed to another student, Eric. The teacher gestured Brad it was now Eric's turn. Apparently there was

something about the statement "It says how much money did she earn?" which Brad felt was clear and strong evidence to support his interpretation.

Eric: It says that she, she sold it for three dollars and forty-nine cents. It says how much did she sell it for, and as I said she sold it for three dollars and forty- nine cents, and then it says how much did she sell it for.

Teacher: Is that what it says? The last sentence says "How much money did she--earn (filled in by students) by selling a plant." Diana.

Eric's explanation gave the first clear reason why some students felt the answer should be three dollars and forty-nine cents. In Eric's current schema, words like "earn" and "sell" were not differentiated. The teacher was puzzled, she wondered if Eric did not "read" the problem correctly. Yet the teacher did not address this question to Eric, but rather to the class. She wanted to hear other students' opinions.

Diana: It says in the end " How much-.

Teacher: Diana, stop for a minute. Wait until everybody settles down (5 seconds pause) Listen.

The teacher used gestures to indicate "Diana, wait." After the class settled down, she gestured to Diana, "Now you can continue."

Diana: It says "How much money did she earn by selling a plant.?" She would have earned three dollars and forty-nine cents because she sold it for three dollars and forty-nine cents. And they don't say, You are not supposed to add two dollars and seventy-five cents and three dollars and forty-nine cents, so she earned three dollars and forty-nine cents.

It was not clear whether Diana really heard Jeff's explanation as he was trying to add those numbers together or she said it unintentionally. Two other students expressed their views on this issue. These two students had basically the same argument as Diana, pointing out, " She would have earned three dollars and forty-nine cents because she sold it for three dollars and forty-nine cents." Yet they also elaborated on Brad's idea of "selling a plant to a friend" They also emphasized that it was impossible to walk out of a store with a plant without paying for it. They drew houses on the board and pointed to "the store" when they explained. Then Hillary went to the board and gave an explanation which surprised the class.

Hillary: I don't think it's um, um, three dollars and forty-nine cents. See, um, I disagree with Jonathan too. Um, because, see when you buy something you don't earn the money that you just gave to the person. Because I meant that was your money, and you didn't just, you didn't just earn the money that you paid for the plant, so I took away, I took away, um, how much the plant cost, because I thought that when you buy a plant, you don't, and then when you get the money back from the plant, because three dollars and forty-nine cents is more than the plant costs by itself. It's just that I thought when you pay for something, you don't earn the money you just pay for --

Jenny: What's your answer?

Hillary: Sixty four cents.

Students: What?

Teacher: Sh--

Hillary: What I am getting is, when you pay for something then, you, you don't, you can earn the money back, but it's usually for something else, So when she bought the plant, then she seld (Students: Sold.) She sold it and then, then she couldn't earn the money back because she didn't just earn the money she just paid for it.

Hillary's explanation surprised many students. It was a new argument and a new answer. Students began to talk among themselves in a controlled manner. Hillary was aware but not concerned that many students disagreed with her. After she finished her explanation. She called on Ann to make a comment.

Ann: When you bought the plant that was with your own money, let's say that's from your allowance. When you went to buy a plant with the money from your allowance, okay? That means you have no more that money but a plant, no money, alright? Then you sell the plant you get those three dollars and forty-nine cents, alright? You don't take it away, because you see, if you bought it with your own allowance, that means you have no more money, just a plant, no more money, just a plant, you don't take it away.

Hillary: I see what you are saying. You say you get your money back from your allowance but-

Ann: (interrupted Hillary) No, not exactly get it back. You get rid of it and get a plant, then you sell the plant and get your money.

Hillary first claimed that she disagreed with the previous explanation, then proceed to offer her interpretation. She thought a person could not earn the money she had just paid for. It appeared that Ann understood Hillary's explanation, and she offered her interpretation of treating the buying and selling process as two separated issues. Ann's rejecting of using the word "back" showed that she know that was the key difference between Hillary's and her own interpretation that is, Hillary treated the buying and selling process as one part when calculating the earning.

The teacher felt that the exchange between Hillary and Ann was very fruitful. However, she felt it was also important for other students' opinions to be heard. Therefore, she stopped them and asked other students to express their views to Hillary's and Ann's interpretations. After heard Jenny's statement, "Because it said how much did she earn. It tells the answer three dollars and forty-nine cents. Because that's what she sold it for. That's what she got. That's why I think it is three dollars and forty-nine cents.", the teacher initiated the discussion on what the word "earn" meant. Many students agreed that by changing the wording from "earn" to "get" did not change the meaning of this problem, but it would if the task asked for the "difference." At this point Eric used an example of a boy earning money by mowing a neighbor's lawn to support his argument that the girl should earn three dollars and forty-nine by selling the plant (emphasized). The teacher began to realize that the word "earn" was normally associated with the money a person got from his service, it only remotely linked to the idea of subtracting the cost. Therefore, she talked about the cost of plants which the class was planning to sell before Mother's Day. She reminded students that they did not get all the plants and potting materials free. The teacher had already paid one hundred dollars out of her pocket for those expenses. The teacher asked students to think about her situation if the class got three hundred dollars from the plant sale. Melvin said, "So mostly you are trying to say is we only get two hundred dollars and you get your one hundred dollars." Still Eric interpreted this as the teacher "had earned a hundred dollars" from the plant sale. At the end, the teacher still did not say that those students who thought they earned all the money they took in from the plant sale were wrong, she just said it depended on how students conceptualized the concept of earn.

Discussion

Because of the focus of this paper and the limitation of the space, we did not provide complete transcripts of these two lessons. In order to get a sense of the overall structure of these two lessons, we offer the following quantitative analysis before we begin the discussion. This analyses involved five variables, number of student participants, number of student/teacher statements and the average length (in words) of each student/teacher statement. A student was counted as a participant only if 1) he or she had permission to speak, or 2) his or her statement was acknowledged by the teacher or the student who at that time had the official speaking right. Only statements made by participants were included in the analysis. We also decided that we would not count utterances and apparent repeats, although we counted transition words like "then", "okay" and "so"...etc. We also did not include any statements made in unison in our analysis because of the difficulty of identifying speakers.

Table 1

	September Lesson	April Lesson
Num. of student participant	7	13
Num. of student statement	30	19
Ave. word length	15.6	58.6
Num. of teacher statement	9	17
Ave. word length	48.9	44.0

Ave. W: average number of words

Table 1 shows that more students participated in the April discussion and each student explanation was longer than those in the September discussion. We found four characteristics of student explanations in the second lesson which were not found in the first lesson. First, the task was re-stated in students' own words. For example, Brad added the details "went back home, and then one of her friends wanted this plant..." to make the original task closer to his ideas of a real life situation; John drew a house on the blackboard to represent the plant shop; Ann talked about using allowance to to buy the plant; Eric raised the issue of earning money by mowing the lawn for neighbors. Second, the answer and computation procedures were no longer the focus of explanations. Even for the students who thought the task was to find the difference between two

dollars and seventy-five cents and three dollars and forty-nine cents, the intent of their explanation was to explain why they thought their interpretation was valid rather than how they found the difference between those two numbers. Third, there was no sense of constraints in terms of how much detail they should put in their explanations. Some speakers would emphasize same points repeatedly in slightly different sequences and wordings. Fourth, all explanations were presented with confidence and enthusiasm, and there was an attempt to give meaning to others' interpretations.

These characteristics showed the growth of student reflection on their problem solving activities and re-presenting their thought processes in words. Barnes (1976) argued that although much learning can happen when students are physically involved in doing or observing, but "Learning of this kind may never progress beyond manual skills accompanied by slippery intuitions unless the learners themselves have an opportunity to go back over such experience and represent it to themselves." (p. 31). Our analysis showed that discussion provided students with such opportunities. In September, potential learning opportunities arose from the attempt to act accordingly to the presumed social norms. So the problematic situations existed when speakers tried to "explain" how they solved the task, what should be included in an explanation was negotiated, and students attempted to understand another person's view point. These problematic situations had the potential of encouraging students to reflect on their own actions, thus restructuring their existing schema. It appeared that the potential learning opportunities in the April discussion arose more from interpreting various view points of a single task than negotiating social norms. The class time spent on negotiating social norms established conditions for students to learn mathematics.

Table 1 also showed that the teacher spoke more in the April lesson than the September lesson. Therefore, it could be argued that the increase of student participation was a result of teacher intervention. Our analysis showed that was not the case because for the following two reasons. First, even though teacher intervention accompanied an increase the number of student's participating, it could not account for the more elaborated, thoughtful student statements as

described in the previous paragraph. Second, the stance the teacher took in these two lessons was different. In the first part of the September lesson, the teacher's role was as a mediator who assisted the negotiation of appropriate interactions and the important aspects of student explanations. In the second part of this lesson, the teacher first acted as a student who tried to figure out what others were saying when June and Jenny had their long argument. Then she felt she knew the source of confusion. She spoke as an authority and acted as though all students would understand after she explained the function of the newspaper company. In April, the teacher's role in the first part was as a facilitator. She used gestures to intervene as soon as private conversations began to interfere with the on-going discussion. She did not wait until the discussion was out of control as she did in September. There was no need to negotiate what was expected, gestures were sufficient to convey messages in many cases. There were quick transitions from one speaker to another, yet speakers were given ample time to explain their viewpoints. The teacher made a special effort to make sure 1) more students were involved and 2) new ideas and different aspects were introduced. So even though the teacher felt that the exchange between Hillary and Ann (as described above) was fruitful, she stopped them and asked other students to express their views. She repeatedly emphasized "Add to that" and reminded students to listen and think what this task meant and what each student was trying to say. Also she constructed an explanation of student's interpretations. She suspected that her students had different interpretations of "earn". So she asked students questions and she found out that many students conceptualized "earn" the same as "get" in this situation. Eric's example helped the teacher realize that the idea of cost seemed to be remote to some of her students. The teacher realized that the idea of earning was complicated, and she felt comfortable leaving this topic without reaching consensus. The teacher commented, "Even some adults would truly believe they "own" the money they spend on credit cards. " The teacher did not see any value in forcing students to accept a mathematical version of "earn" which turned out to be socially over-simplified.

It appeared that the teacher was no more an authority in April than in September, although she played a much more active role in April. Private discussions, talking out of turn, aggressive

attitudes still existed. Yet none of those became unmanageable so as to interrupt the discussion. Students also showed more awareness of the social norms in April, and it was clear from their statements that they had heard and understood each other although the might not agree. So far we have presented partial transcripts of two lessons. We have also characterized the qualitative and quantitative differences between them. In the following sections, we will focus on two issues, 1) What factors contributed to the observed changes? and 2) how might our findings contribute to mathematics education.

Looking Back over the School Year

Intellectual autonomy was one of the teacher's goals throughout the year. She wanted students to take responsibility for making reasonable judgments. At the beginning of the school year, the teacher gave her students both the responsibility of managing turn taking and determining the viability of an interpretation. Usually, she sat at the back of the classroom and let the speakers be in charge as long as the discussion proceeded smoothly. In no case did she evaluate student answers or choices of solution methods. Even when students could not reach a consensus on a task after a discussion for fifteen minutes, she would still not resolve the conflict by taking a position on the dispute. Instead she would post the task on the board and encourage students to think more about it. The teacher did not feel that a discussion was productive only if students sat quietly and followed rules of speaking. She tried to encourage students to express their ideas openly without being restricted by a rigid structure of exchange.

Achieving this goal was not an easy task for the students and the teacher. Situations like students accusing each other of violating rules and ineffective communication were typical instances at the beginning of the school year. At first, the teacher felt that the situation would be improved if speakers would give "clear" explanations. So the class had several discussions on "What can a sharer could do to help us understand?" Suggestions like, speak up, face the audience while talking, stand aside so other students can see the board, and tell the answer first did improve the physical part of the presentation, yet did not necessarily enhance communication. But suggestions like, be prepared, explain, and tell what you did were not simple matters. Several

instances indicated that students gave different meanings to what should be included in their explanations, and they tended to think what was clear to them would also be clear to other students. The emphasis on "clear" explanations gave some students excuses to blame the speaker for their lack of understanding. They became impatient, and accused the speaker of not being honest when they could not make sense of the explanation immediately. An exchange on September 27, during which Mary was intimitated by several students yelling at her for her insisting on one answer yet was unable to give a "clear"explanation, changed the teacher's attitude. The teacher began to take a more active role in protecting speakers from interruptions, and assisting them in expressing their ideas.

The teacher faced a dilemma when considering the individual needs and smooth group functioning. She could not publicly encourage students who talked without first raising their hands, yet some of those spontaneous student responses were insightful and exactly what she thought would solve the issue at hand. She was more interested in listening and figuring out her students' ideas than managing their actions. Yet she also noticed that some of her students (not all of them) would be distracted if many talked at the same time. For the benefit of those students, she asked the class to be quiet and to sit still. She realized that the more she talked, the less time would be available for students to express their thoughts. Even though she had no doubt that her students were capable of asking the same questions or explaining the situations as she did; she never knew what a particular student might say at a particular moment. The teacher did not have predetermined strategies to cope with these situations. She strove for a balance between individual and the group as well as short term and long term effects. Sometimes she might not be aware of or be able to explain her decisions, and she was not always the one who determined the nature of class discussions.

Students also felt the psychological and social tensions between being individual learners and members of the class community. Students wanted to have opportunities to explain their own methods, to ask questions, comment on another student's explanation, and to defend their own explanations as individual learners. Yet as members of the class community, there were implicit

and explicit rules to follow which might determine when, what and how students could talk (Bauersfeld, 1980, 1988; Bishop, 1985). For example, there was one instance when Raymond was the first person to explain his way of making the number fifty. He was excited and he did a nice job of explaining. As he watched the second student gave a different way to make fifty, he thought of another good way. He tried to get a second opportunity to share but failed to do so, because other students also wanted to share. He was unhappyeach time; he was not recognized by the teacher. At that time some students who had learned the principle of fair chance, so they would not attempt to share after they had had their chance. This example also showed how social factors might have affective influences on individual learners.

One major challenge the class faced was to communicate mathematical ideas when there was a disagreement on individual expectations of an explanation or justification. In the September lesson, June first indicated that she did not understand Jenny's explanation. Because Jenny attempted to elaborate on her previous statement, a situation for explanation was interactively constituted by Jenny and June (Cobb, et.al, in press). However, Jenny and June could not agree on the legitimacy of the justification which Jenny offered to justify her action of taking the eighteen cents away. June's statement indicated that she thought Jenny had not explained; Jenny's statement indicated that she thought June was not listening, while they both thought they had tried communicated to the other person. Emotions were aroused by the communication breakdown and interfered with future efforts to communicate. On the first day of the second semester, the class had a discussion about what they liked and disliked about class discussion. Many students said they did not like the interruptions and yelling when there was an disagreement. In subsequent lessons, the teacher would remind students of these things. For example, the teacher would say to students who interrupted the discussion or those who were about to do so, "You said you don't like to be interrupted." Later the teacher identified that particular discussion as a beginning point of more student self-control as was shown in the April lesson. Only after the social norms had been negotiated, were they able to talk mathematics effectively.

Conclusion

In this paper, we have compared and contrasted the interactive patterns and potential learning opportunities of two class discussions to show the growth the class made during the school year. We also described the psychological and social tensions the teacher and students faced as they attempted to talk mathematics. Learning to talk mathematics was not an easy task for the class we studied. Before they could discuss mathematics, they had to learn how to talk mathematics as a group. In September a discussion would become an argument between a few students because students had not learned how to communicate their mathematics ideas effectively. As students interacted with each other, they gradually realized that it was not enough to just disagree with someone else's view, they had to present their reasons for disagreeing. In April after the class had interacted with each other in the same format with the same purpose for five months, they had negotiated a set of social norms and taken-to-be-shared mathematics meanings which enabled them to interact with each other talking mathematics.

Although like Erikson, we wondered about the fine line between chaos and discussion in many situations (referenced by Cazden, 1988); we believe that a certain degree of "chaos" is essential in the process of establishing taken-to-be-shared norms. These experiences helped students anticipate the consequences of their own actions in relation to the whole group. Even though the teacher felt that these changes were the result of more experience and maturity, we believe that students' desire to present and discuss their ideas with other people, taking responsibility of making mathematics decisions and the teacher maintaining a non-judgemental role were important in explaining these changes.

Implications

The data presented in this paper suggested that class discussion is a rich environment for mathematics learning. Student growth could be seen from the perspectives of individual learners and members of the class community. As individual learners, students had more elaborated and integrated thinking schema and greater awareness of their own thought processes. They also were able to make sense and communicate their mathematical ideas more effectively. As members of the

class community, students learned the need to negotiate mathematical meaning and social norms, to respect other people's right of speech, to control emotions resulting from communication failure, and to act according to the taken-to-be-shared social norms. It is important for classroom teachers to use class discussion in mathematics instructions, so students can have opportunities to experience "mathematics as problem solving", "mathematics as communication", and "mathematics as reasoning."

Barnes (1976) argued that language is an integral part of the learning process and not just words which convey meaning. Thus we must recognize that students are active participants in the making of meaning rather than passive recipients of socialization (p.31). Our data further suggests that students can't negotiate mathematics meaning without first negotiating social norms. A particular set of social norms are negotiated in each class for the purpose of talking mathematics. We hope an account of a class's attempt on "learning to talk mathematics" can provide insights for both mathematics educators and classroom teachers to reflect on current practices in mathematics education.

References

Barnes, D. (1976). From communication to curriculum. London: Penguin Books Ltd.

Balacheff (1990). Towards a problematique for research on mathematics teaching. Journal for Research in Mathematics Education 21(4), 258-272.

Bauersfeld, H. (1980) Hidden dimensions in the so-called reality of a mathematics classroom. Educational Studies in Mathematics, 11, 23-41.

Bauersfeld, H. (1988). Interaction, construction, and knowledge: Alternative perspectives for mathematics education. In T. Cooney & D. Grouws (Eds.), Effective mathematics teaching (pp.27-46). Reston, VA: National Council of Teachers of Mathematics; Hillsdale, NJ: Lawrence Erlbaum Associates.

Bishop, A. (1985). The social construction of meaning-a significant development for mathematics education? For the Learning of Mathematics, 5, (1), 24-28.

Bruner, J. S. (1968). Actual Mines, possible worlds. Cambridge, MA: Harvard University

Press.

Cazden , B. C. (1988). Classroom discourse: The language of teaching and learning. Portsmouth, NH: Heinemann.

Cobb. P. (in press). Reconstructing elementary school mathematics. Focus on Learning Problems in Mathematics.

Cobb, P., Wood, T., Yackel, E., Nicholls, J.,.Wheatley, G., Trigatti, B., & Perlwitz M. (1991). Problem-centered mathematics projects. Journal for Research in Mathematics Education, 22, (1), 3-29.

Cobb, P., Wood, T. Yackel, E. & NcNeal, B. (in press) Characteristics of classroom mathematics tradition: An interactional analysis. In C. Maher & R. Davis (Eds.), Relating schools to the reality of mathematics learning. Englewood Cliffs, NJ: Prentice Hall.

Duckworth, E. (1987). The having of wonderful ideas. New York, NY: Teacher College Press...

Lo. J., Wheatley, G. & Smith A. (1990). Potential mathematics learning opportunities in grade three class discussions. In G. Booker, P. Cobb & T. N.de Mendicuti (Eds.) The Proceedings of the Fourteenth International Group for the Psychology of Mathematics Education (121-128).

National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: National Council of Teachers of Mathematics.

Notional Research Council. (1989). Everybody counts: A report to the nation on the future of mathematics education. Washington, D. C.: National Academy Press.

Pirie S. E. B. & Schwarzenberger R. L. E.(1988). Mathematical discussion and mathematical understanding. Educational Studies in Mathematics, 19, 450-470.

Schoenfeld, A. H. (1988). Problem solving in context (s). In R. I. Charles & E. A. Silver, (Eds.), Teaching and assessing of mathematical problem solving (pp.82-92). Reston, VA: National Council of Teachers of Mathematics and Hillsdale, NJ: Lawrence Erlbaum Associates.

Smith, D. (1987). It's not what you say, it's the way that you do it. Mathematics Teaching, 121,

7-11.

Sless, D. (1986). In search of semiotics. Beckenham: Groom Helm Ltd.

Solomon Y. (1989). The Practice of Mathematics. London: Routledge

Steffe, L. (1990). Overview of the Action Group A1: Early Childhood years. In L. Steffe & T. Wood (Eds.) Transforming children's mathematics education: International perspectives. (pp.3-15). Hillsdale, NJ: Lawrence Erlbaum Associates.

von Glasersfeld, E. (1989). Cognition, construction of knowledge, and teaching. Synthese, 80. 121-140.

Yackel, E., Cobb, P. & Wood, T. (in press). Small group interactions as a source of learning opportunities in second grade mathematics. Journal for Research in Mathematics Education Monograph.